A REVIEW OF MATH OPERATIONS

It took the human race centuries to develope mathematical concepts which are considered simple by today's standards. Developing a number system to accommodate the ever-growing needs of man was a long, hard struggle. There were some interesting milestones on this road of progress.

One of these milestones was the idea of number POSITION and PLACE VALUE. In the number, 333, the three at the right means three things or three units. The middle three means three times ten or 30, and the three to the left means 3 times one hundred or 300. The discovery of place value gave civilization a terrific spurt, for with this system, it was possible to write any number however large or small.

The idea of ZERO on our number scale was another milestone on the road to progress. In fact, zero has been called one of man's greatest inventions. This statement is not as fantastic as it sounds. If it were not for zero, numbers such as 100 would be difficult to indicate.

59-1. Addition

When adding WHOLE NUMBERS such as 1, 2, 3, etc., they must be written according to our place value system - units to units, tens to tens, etc. The tens are added to tens, hundreds to hundreds, etc.

To ADD, begin at the top of the units column and add down. The units column may be found in the example problem.

EXAMPLE:

```
 ┌─────────thousands
 │ ┌───────hundreds
 │ │ ┌─────tens
 5,749 ←───units
   123
     5
14,657
──────
20,534
```

It is often helpful to mentally add the numbers as one is progressing down through the column. The sum of the units column is 24. The number, 24, is composed of 4 units and 2 tens. Therefore, a 4 placed in the units column below the line, and a 2 is "CARRIED OVER" to the tens column where it will be added to the numbers in the tens column. The tens column is then added. The sum of the tens column (including the 2 which was "carried over") is 13. The 3 is written in the tens column, and the 1 is "carried over" to the hundreds column. This process is continued till all of the numbers in each column are added.

EXERCISE 1:

Add the numbers in the following problems:

```
1.   631        2.   68        3.    462        4.  4,379
     222            723              321               323
      31             11            8,921               182

5. 7,221,692    6. 3,256       7.    889        8.    218
     341,222       2,445           3,654              666

9.  60,000     10.  756
     3,500          234
                    785
```

59-2. Subtraction

SUBTRACTION is the operation of finding the difference between two numbers. This is the same as finding the amount that must be added to one number, called the SUBTRAHEND, to equal another number, called the MINUEND.

In subtraction, as in addition, the units must be placed under the units, the tens under the tens, etc.

EXAMPLE:

Subtract 684 from 992.

992 is equal to 9 hundreds, 9 tens, and 2 units

684 is equal to 6 hundreds, 8 tens, and 4 units

Since four units cannot be subtracted from two units, one tens value is "BORROWED" from the tens column and added to the units column.

Therefore:

992 is equal to	9 hundreds,	8 tens,	12 units
684 is equal to	6 hundreds,	8 tens,	4 units
308	3 hundreds,	0 tens,	8 units

Hence, 684 subtracted from 992 is equal to 308.

EXERCISE 2:

Subtract the following:

$$\begin{array}{llll}
1.\ \ \begin{array}{r} 42 \\ \underline{33} \end{array} & 2.\ \ \begin{array}{r} 683 \\ \underline{672} \end{array} & 3.\ \ \begin{array}{r} 6,011 \\ \underline{2,133} \end{array} & 4.\ \ \begin{array}{r} 564 \\ \underline{223} \end{array} \\
5.\ \ \begin{array}{r} 49 \\ \underline{26} \end{array} & 6.\ \ \begin{array}{r} 786 \\ \underline{427} \end{array} & 7.\ \ \begin{array}{r} 831 \\ \underline{165} \end{array} & 8.\ \ \begin{array}{r} 322 \\ \underline{231} \end{array} \\
9.\ \ \begin{array}{r} 888 \\ \underline{349} \end{array} & 10.\ \ \begin{array}{r} 781 \\ \underline{392} \end{array} & &
\end{array}$$

59-3. Multiplication

MULTIPLICATION is defined as the operation of adding a number to itself a given number of times. Therefore, 4 X 8 (four times eight) could be thought of as adding 8 four times.

The number that is to be multiplied is called the MULTIPLICAND, and the number of times it is to be added is called the MULTIPLIER. The answer obtained from performing the operation of multiplication is called the PRODUCT.

EXAMPLE:

6 times 3 means 6 + 6 + 6 equals 18

$$\begin{array}{rl} 6 & \text{multiplicand} \\ \times\ \underline{3} & \text{multiplier} \\ 18 & \text{product} \end{array}$$

To multiply the number 683 by 4, place the multiplier under the multiplicand so that the units will be under the units, the tens under the tens, etc.

$$\begin{array}{r} \underline{1}\ \, \\ 683 \\ \underline{4} \\ 2 \end{array} \qquad\qquad \begin{array}{r} \underline{31}\ \\ 683 \\ \underline{4} \\ 2,732 \end{array}$$

The operation of multiplication is performed as follows: Begin at the right by multiplying the unit digit of the multiplicand by the unit digit of the multiplier. Three units times four units is equal to 12 units which is equivalent to 1 tens and 2 units. Therefore, write the 2 in the units column of the product and "CARRY" the one to the tens column of the multiplicand where it will be added to the tens column of the product after the tens column of the multiplicand is multiplied by the multiplier.

Next, multiply the 4 times the 8 and add the carried one, 4 x 8 = 32. Adding the one 32 + 1 = 33 which is 3 hundreds and three tens. Place the three tens in the tens column of the product and carry the 3 hundred to the hundreds column of the multiplicand.

Repeating the same process, multiply the 4 times the 6 and the carried 3 is added to that product (4 x 6) + 3 = 27. This figure, 27, is equal to 2 thousands, and 7 hundreds. Place the 7 in the hundreds column of the product, and carry the 2. Since this operation completes the multiplication, the 2 should be placed in the thousands column of the product. Therefore, the product of 683 times 4 is equal to 2,732.

EXERCISE 3:

Perform the following operations:

1. 36 x5	2. 40 x4	3. 95 x2	4. 87 x3
5. 536 x4	6. 3,467 x5	7. 48 x86	8. 453 x364

59-4. Division

DIVISION is the operation of determining how many times a given number, called a DIVIDEND, contains another number called a DIVISOR. Division may also be stated as an operation of repeated subtractions. The resultant, or answer, is called a QUOTIENT. The signs which indicate division are:

$$\div \qquad — \qquad / \qquad \overline{)}$$

Therefore, 6 divided by 3 may be written as $6 \div 3$, $\frac{6}{3}$, $6/3$, or $3\overline{)6}$.

A simple procedure for dividing is as follows:

Step One:

To find the quotient of 47.9 divided by 7.24, place the dividend under the line of the division sign. Place the divisor before the division sign.

$$7.24\overline{)47.9}$$

Step Two:

Move the decimal point in the divisor to the right of the divisor's extreme right digit counting the number of places moved. Move the decimal point of the dividend to the right the same number of places. Use zeros to fill the places at which the dividend has no value. Place a decimal point above the line over the decimal point which was moved in the dividend.

$$7{\times}24.\overline{)47{\times}90.}$$

Step Three:

Determine the number of times the divisor will go into the dividend. Start by seeing if the divisor will go into the digit (4). Since it will not, try the digits (47). Keep this up until the divisor will go at least one time. 724 will go into 4790 approximately 6 times. Place the 6 over the line above the last digit of the dividend that was used in this step. Multiply the 6 by the divisor and subtract the product from the digits of the dividend used in this step. This difference should be less than the divisor. If not, increase the number in the quotient by one.

```
        6.
724/4790.
    4344
     446
```

Step Four:

The next digit in the dividend is understood to be zero. Therefore, this zero is brought down and placed next to the difference obtained in step three. Determine how many times 724 can be divided into 4460. Place this number, 6, in the quotient to the right of the decimal point. Multiply the multiplier by this number and subtract the product from 4460 obtaining the new difference of 116.

```
        6.6
724/4790.0
    4344
     4460
```

The process of separating a number into two or more smaller numbers having the original number as their product is called FACTORING. The smaller numbers obtained are called FACTORS of the larger number. In the last example, 6/8, the number 2 was a common factor of both the numerator and the denominator.

EXAMPLE:

$$\frac{6}{8} = \frac{3 \times 2}{4 \times 2} = \frac{3}{4} \times \frac{2}{2} = \frac{3}{4} \times 1 = \frac{3}{4}$$

Factors common to both numerator and denominator can and should be divided out.

When reducing a fraction to its lowest term, find the common factors in both the numerator and denominator and divide them out.

EXAMPLE:

$$\frac{9}{21} = \frac{3 \times 3}{3 \times 7} = \frac{3}{7}$$

Notice that there are two three's in the numerator of the previous example, and only one in the denominator. Only one pair may be divided. The quotient of the division is one. Multiplying one by any quantity is equal to the number.

59-10. Prime Factors

As previously defined, a factor of a whole number is any whole number which will divide into the whole number evenly. Thus, 2, 5 and 7 are factors of the number 70. The number one is also a factor. However, it is not normally shown. It is assumed to be present.

A PRIME NUMBER is a number which has been factored to the point where it can only be divided by itself and the number one. The numbers 2, 5, 7 and 1? are examples of prime numbers.

Utilizing the definitions of factors and prime numbers, it follows that the prime factors of a whole number is any prime number that will divide the whole number evenly. Thus, the numbers 3, 5 and 7 are prime factors of the number 105.

L...ing the prime factors of a number is the process of finding the prime numbers that will ex-...ly divide into that number. Begin this process by dividing the number by the smallest prime number 2, and continue dividing the consecutive quotients by 2 until it will not divide into the quotient evenly. Then divide the quotient by successively higher prime numbers, and continue this process until the final quotient is the number one.

EXAMPLE:

Find the prime factors of the number 60.

```
2 /60
2  /30
3   /15
5    /5
      1
```

Therefore, the number 60, can be expressed as a product of its prime factors.

$$60 = 2 \times 2 \times 3 \times 5$$

EXERCISE 6:

Find the prime factors of the following numbers:

1. 420 2. 780 3. 12 4. 36 5. 75

59-11. Lowest Common Multiple

A MULTIPLE of a whole number is a whole number which will divide the original whole number evenly. Thus, 36 is a multiple of the number 6, and the number 24 is a multiple of the number 8.

...a whole number can be divided evenly by two or more numbers, it is a COMMON MULTIPLE. ..., the number 36, is a common multiple of the numbers 6 and 3. The smallest number which ...be divided evenly by two or more numbers is the LOWEST COMMON MULTIPLE, and is abbreviated L.C.M. The number 15 is the L.C.M. of the numbers 3 and 5.

To find the L.C.M. of two or more numbers, factor each number into its prime factors and find the product of all the different prime factors using each different factor the greatest number of times it appears in any one number.

Step Five:

Repeat step four two more times thereby carrying the division out three places which will be considered sufficiently accurate for most work in electricity. If the value of the final remainder is equal to half of the divisor or more, increase the last digit by one. This is called "ROUNDING UP". If the difference is less than half do not change the quotient (round down). If extreme accuracy is required, indicate that the final difference is the remainder 116. Always check the final answer by multiplying the quotient by the divisor and adding the final difference of 116.

EXERCISE 4:

Performing the following operations: (carry out to two significant figures)

1. $645 \div 9$ 2. 8 249 3. $\frac{289}{4}$ 4. $\frac{100}{37}$

59-5. Arithmetic Mean

Many times, it is required to apply a combination of operations to find the ARITHMETIC MEANS of voltages, currents and powers. The arithmetic mean is a simple average of a group of numbers. In fact, it is often called the AVERAGE. Suppose you wish to determine the average value of your blitz grades. The sum of all of the blitz grades divided by the total number of individual grades considered will render the mean blitz grade.

EXAMPLE:

Find the arithmetic mean of the following numbers: 100, 90, 70, 80, 70, 90, and 80.

Step One:

Find the sum of the numbers.

$$\begin{array}{r} 100 \\ 90 \\ 70 \\ 80 \\ 70 \\ 90 \\ \underline{80} \\ 580 \end{array}$$

Step Two:

Divide this sum by the number of blitzes.

$$\begin{array}{r} 82.857 \\ 7\overline{)580.000} \\ \underline{56} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 50 \end{array}$$

The average or mean of the grades is 82.857. Rounding off, the average grade is 82.86.

COMMON FRACTIONS

59-6. Definitions

A COMMON FRACTION is an indicated division, it expresses a number of equal parts into which something has been divided. As an example, 3/8 could be thought of as some object that has been divided into 3 of 8 equal parts, or as a division, 3 divided by 8.

The number under the line is called the DENOMINATOR. It shows the number of parts into which the object has been divided. The number above the line is called the NUMERATOR. It tells how many parts are taken or considered.

There are two types of fractions - PROPER and IMPROPER. A proper fraction is a fraction the numerator of which has a smaller value than the denominator. As an example, 1/2 and 3/4 are proper fractions. An improper fraction is one in which the numerator is equal to or larger than the denominator. As an example, 4/4, and 9/8 are improper fractions.

59-7. Properties of Fractions

An important principle involving fractions is that: The numerator and denominator of any fraction can be multiplied or divided by the same factor (excluding zero) without changing the value of the fraction. This is true because the number one (sometimes called UNITY) is the identity element for the operation of multiplication and division. This means that if a quantity, such as six, is multiplied or divided by one, it will not change the value of the quantity. Six times one is equal to six. Six divided by one is equal to six. Therefore, if the fraction 2/3 is multiplied by the fraction 4/4 (which is equal to one), the value of the fraction will be unchanged.

EXAMPLE:

$$2/3 \times 4/4 = 8/12$$

$$12/16 = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$$

59-8. Changing a Mixed or Whole Number to an Improper Fraction

By definition, a WHOLE number is a number which contains no fractions, and a MIXED NUMBER is one which contains a whole number plus a fraction. The numbers 2, 4 and 6 are whole numbers. The numbers $2\frac{1}{2}$ and $3\frac{1}{4}$ are mixed numbers.

When performing computations with whole or mixed numbers, it is sometimes necessary to change them into improper fractions. The following examples will illustrate the process involved.

EXAMPLE:

Change the number 5 to a fraction which has the number 6 in the denominator.

$$\text{since } 6/6 = 1$$

$$\text{then } 5 \times \frac{6}{6} = \frac{30}{6}$$

Change the mixed number $3\frac{1}{2}$ to a fraction which has a numerical denominator of 2.

$$\text{since } 3\frac{1}{2} = 3 + \frac{1}{2} \qquad \text{then } 3\frac{1}{2} = 3 \times \frac{2}{2} + \frac{1}{2}$$

$$\text{and } \frac{2}{2} = 1 \qquad = \frac{6}{2} + \frac{1}{2} = \frac{7}{2}$$

This process may be summarized by the following statements:

To change a whole number to an improper fraction, multiply the whole number by a unit fraction (fraction equal to one) with the desired denominator.

To change a mixed number to an improper fraction, change the whole number part of an improper fraction having the desired denominator, and then add the fraction.

EXERCISE 5:

1. Convert the following to a fraction which has the number nine in the denominator:

a) 5 b) 10 c) 15 d) 32 e) 18

2. Change the following to improper fractions.

a) $2\frac{1}{3}$ b) $3\frac{3}{4}$ c) $6\frac{1}{7}$ d) $62\frac{1}{8}$ e) $7\frac{1}{3}$

59-9. Reducing a Fraction to its Lowest Terms

Fractions reduced to their lowest terms are easier to read, have more meaning, and are much easier to work with. For these reasons, fractions are always reduced to their lowest terms.

To reduce fractions to their lowest terms, the numerator and the denominator are searched for a common number which can be divided evenly into both numbers. As an example, in the fraction 6/8, both the numerator and the denominator may be divided by 2 resulting in:

EXAMPLE:

Find the L.C.M. of the numbers 8, 12, and 24.

Factoring into prime factors:

$$8 = 2 \times 2 \times 2$$

$$12 = 2 \times 2 \times 3$$

$$24 = 2 \times 2 \times 2 \times 3$$

The different prime factors are the numbers 2 and 3. The number 2 appears twice as the prime factor of 12, three times as the prime factor of the number 8, and three times as the prime factor of the number 24. The number 3 appears once as a prime factor of 12 and once as a prime factor of 24. Therefore, to find the L.C.M. of the group of numbers, the number 2 must be used as a factor three times and the number 3 must be used as a factor once.

Hence: L.C.M. $= 2 \times 2 \times 2 \times 3$

$= 24$

Find the L.C.M. of the numbers 8, 36, and 15.
Expressing each number in its prime factors:

$$8 = 2 \times 2 \times 2$$

$$36 = 2 \times 2 \times 3 \times 3$$

$$15 = 3 \times 5$$

Therefore, the L.C.M. is:

L.C.M. $= 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$= 360$

EXERCISE 7:

Find the L.C.M. of the following:

1. 10, 15 and 30
2. 9, 24 and 36
3. 12, 18 and 32
4. 6, 105 and 8
5. 20, 30 and 60

59-12. Lowest Common Denominator

The LOWEST COMMON DENOMINATOR of two or more fractions is the lowest number that is exactly divisible by the given denominators which is the L.C.M. of the denominator. The lowest common denominator is abbreviated L.C.D.

The following steps may be applied in reducing fractions to their L.C.D.'s.

Step One:

Find the L.C.D. of the denominator. This is the same process as finding the L.C.M. of a group of numbers.

Step Two:

Using each fraction - one at a time - divide the L.C.D. by the denominator of the fraction considered, and multiply both numerator and denominator of the fraction considered by the quotient thus obtained.

EXAMPLE:

Reduce $\frac{1}{6}$, $\frac{1}{9}$ and $\frac{1}{13}$ to their L.C.D.

L.C.D. = L.C.M.

The L.C.M. of the numbers 6, 9 and 18 is:

L.C.M. = 18

Changing the fractions to the lowest common denominator, 18.

$$\frac{1}{6} = \frac{1}{6} \times \frac{3}{3} = \frac{3}{18}$$

$$\frac{1}{9} = \frac{1}{9} \times \frac{2}{2} = \frac{2}{18}$$

$$\frac{1}{18} = \frac{1}{18} \times \frac{1}{1} = \frac{1}{18}$$

Reduce $\frac{1}{12}$, $\frac{1}{18}$, $\frac{1}{36}$

L.C.M. = L.C.D.

L.C.D. = 36

$$\frac{1}{12} = \frac{1}{12} \times \frac{3}{3} = \frac{3}{36}$$

$$\frac{1}{18} = \frac{1}{18} \times \frac{2}{2} = \frac{2}{36}$$

$$\frac{1}{36} = \frac{1}{36} \times \frac{1}{1} = \frac{1}{36}$$

9-13. Addition and Subtraction of Fraction

Fractions having common denominators are added or subtracted by adding or subtracting the numerators of the fractions considered. Thus, to add two or more fractions having common denominators, add the numerators and write the sum over the common denominator.

EXAMPLE:

$$\frac{2}{8} + \frac{3}{8} = \frac{2+3}{8} = \frac{5}{8}$$

When subtracting two fractions, subtract the numerator of the subtrahend from the numerator of the minuend and write the difference over the common denominator.

EXAMPLE:

$$\frac{5}{8} - \frac{3}{8} = \frac{5-3}{8} = \frac{2}{8} = \frac{1}{4}$$

When performing the operations of addition and subtraction on fractions having unlike denominators, observe the following rules:

Change the fractions to equivalent fractions having a L. C. D.

Perform the indicated operation on the numerators of the equivalent fractions.

EXAMPLE:

$$\frac{1}{8} + \frac{1}{12} + \frac{1}{24} = \frac{3}{24} + \frac{2}{24} + \frac{1}{24} = \frac{3+2+1}{24} = \frac{6}{24} = \frac{1}{4}$$

$$\frac{1}{4} - \frac{1}{8} - \frac{1}{12} = \frac{6}{24} - \frac{3}{24} - \frac{2}{24} = \frac{6-3-2}{24} = \frac{1}{24}$$

EXERCISE 8:

Perform the indicated operations:

1. $1/2 + 1/3$
2. $7/8 + 1/12 + 1/24$
3. $1/4 - 1/8 - 1/12$
4. $4/5 + 2/10 + 4/20$
5. $2\frac{1}{3} + 3\frac{1}{4}$
6. $4/5 + 20/11 + 2/8$
7. $1\frac{1}{3} + \frac{6}{8}$
8. $\frac{1}{2} - \frac{1}{4}$
9. $1/5 - 1/15 - 1/30$
10. $18/32 + 5/4 - 1/5$

59-14. Multiplication of Fractions

To multiply two or more fractions, multiply the numerators by the numerators and the denominators by the denominators. The product of the numerators of the factors becomes the numerator of the product. The product of the denominators of the factors becomes the denominator of the product.

EXAMPLE:

$$2/5 \times 4/5 = \frac{2 \times 4}{5 \times 5} = \frac{8}{25}$$

$$1/2 \times 3/4 \times 7/8 = \frac{1 \times 3 \times 7}{2 \times 4 \times 8} = \frac{21}{64}$$

In the previous example, the order in which the multiplication took place was unimportant. The fraction 3/4 could have been multiplied by 7/8, and that product multiplied by the remaining fraction 1/2. The product would be the same.

EXERCISE 9:

Find the product of the following:

1. $5/8 \times 12$
2. $5 \times 4/9$
3. $1/3 \times 2/3$
4. $1/2 \times 1/3 \times 2/5$
5. $3/4 \times 6$
6. $4/3 \times 1/6$
7. $10\frac{1}{2} \times 3\frac{1}{3}$
8. $1\frac{5}{6} \times 13$
9. $8\frac{3}{4} \times 2/5$

59-15. Division of Fractions

To divide one fraction by another, invert the divisor (the one being divided into the other) and multiply.

EXAMPLE:

$$1/2 \div 1/3 = 1/2 \times 3/1 = 1\frac{1}{2}$$

$$2/5 \div 3/5 = 2/5 \times 5/3 = 2/3$$

$$1\frac{1}{2} \div 2/3 = 3/2 \div 2/3 = 3/2 \times 3/2 = 9/4 = 2\frac{1}{4}$$

Notice that, just as in multiplications, the first step is to change the mixed numbers to improper fractions. The second step is then to invert the divisor. The third step is to multiply, following all of the rules from multiplication.

EXERCISE 10:

Perform the following divisions:

1. $3/8 \div 2/3$
2. $2\frac{1}{2} \div 1\frac{1}{2}$
3. $5/8 \div 5/16$

59-16. Decimal Fractions

The decimal system is a convenient way to write complicated numbers and mixed fractions. Decimals are easier to add, subtract, multiply and divide than fractions. For this reason, it is important to be able to convert from a decimal to a fraction and from a fraction to a decimal.

The word decimal is derived from the Latin word <u>decum</u>, meaning ten. Essentially, decimals are another way of writing fractions having denominators of 10, 100, 1,000, etc. For example, the number 0.3 is a fraction written in the decimal system. It represents the fraction 3/10. The period (.) between the digit 0 and the digit 3 is called a decimal point. The location of the decimal point determines whether the denominator of the fraction it represents should be 10, 100, 1,000, etc.

EXAMPLE:

$$0.3 = 3/10$$

$$0.03 = 3/100$$

$$0.003 = 3/1000$$

$$0.0003 = 3/10,000$$

Whole numbers are written in the decimal system by placing the decimal point after the last digit in the number.

EXAMPLE:

$$3 = 3.0, \quad 60 = 60.0, \quad \text{and} \quad 800 = 800.0$$

Very complicated numbers and mixed fractions can be written very easily in the decimal system. Notice the following decimals and their meanings:

$$1.680 = \frac{1,680}{1,000} = 1\frac{680}{1000}$$

$$100.8 = \frac{1,008}{10.00} = 100\frac{8}{10}$$

$$396.71 = \frac{39,671}{100} = 396\frac{71}{100}$$

$$0.0085 = 85/10,000$$

Memorize the denominators associated with a specific location of the decimal point. Notice also that the number of zeros in the denominator always equals the number of places to the left of the last digit in the decimal. This rule is true even for the decimal version of whole numbers.

EXAMPLE:

$0.52 = 52/100$ (decimal point two places to the left of last digit, two zeros in the denominator.)

$$167.8 = 1,678/10 = 167\frac{8}{10} = 167\frac{4}{5}$$

(decimal point one place to the left of last digit - one zero in the denominator)

59-17. Converting Fractions to Decimals

To convert fractions to decimals, perform the following operations:

Write the numerator as a decimal by adding a decimal point after the last digit.

Divide the denominator into the numerator.

Add the zeros after the decimal in the numerator as necessary.

Place a decimal point in the answer vertically above its location in the numerator.

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Place a decimal point in the answer vertically above its location in the numerator.

EXAMPLE:

$$1\frac{1}{2} = 1 + \frac{1}{2}$$

$$\frac{1}{2} = 2\overline{)1.0}^{\;0.5}$$

Therefore:

$$1\frac{1}{2} = 1 + 0.5 = 1.5$$

$$1\frac{3}{32} = 1 + \frac{3}{32}$$

$$\frac{3}{32} = \begin{array}{r} .09375 \\ 32\overline{)3.00000} \\ \underline{288} \\ 120 \\ \underline{96} \\ 240 \\ \underline{224} \\ 160 \\ \underline{160} \\ 0 \end{array}$$

$$1\frac{3}{32} = 1 + 0.09375$$

$$1\frac{3}{32} = 1.09375$$

It can be seen from the above examples that it is extremely important to do this type of work neatly, so that the decimal point may be located with little difficulty.

Notice that sometimes it makes no difference how many zeros are added to the number being divided. It will still not divide without a remainder. In these situations division is only accomplished to meet the requirements of the problem. Sometimes the required accuracy of an answer may be out to several decimalplaces. In other applications an answer accurate to one decimal place will be sufficient.

59-18. Adding and Subtracting Decimals

When adding and subtracting decimals, be sure that the decimal points are arranged vertically directly over one another. Then add or subtract the number as if they were whole numbers.

To add 60.0, 0.003, 1.6 and 32.05, the numbers are arranged in columns, and the decimal points in each row occupy the same position. This is shown in the example.

EXAMPLE:

$$\begin{array}{r} 60.0 \\ 0.003 \\ 1.6 \\ \underline{32.05} \\ 93.653 \end{array}$$

The same rule holds true for subtraction.

EXAMPLE:

Subtract 0.008 from 2.687

2.687
0.008

EXERCISE 11:

Add the following:

1. 68.4
   32.1

2. 0.004
   0.026

3. 6.01
   0.023

4. (4)+(0.68) + (1.23)

5. (67.9)+(4.52)

59-19. Multiplication and Division of Decimals

When multiplying decimals, multiply as if the decimals were whole numbers. Place the decimal point in the final answer at that place, which is the sum of the decimal places in the factors, to the left of the last digit in the product.

EXAMPLE:

6.0
3.0
18.00 (two places to the left of the last digit)

30.6
0.007
0.2142 (four places to the left of the last digit)

9.3
0.0008
0.00744 (five places to the left of the last digit)

When dividing decimals, move the decimal point in the divisor to the right of the last digit that is zero. Also, move the decimal point in the number being divided the same number of places adding zeros as necessary. The decimal point in the answer should be placed directly above the decimal point (at its final location) in the number divided. Divide as if the numbers were whole numbers.

EXAMPLE:

$$6.8 \div 3.2 = 32\overline{)68.000} = 2.125$$

```
        2.125
32 / 68.000
     64
      40
      32
       80
       64
       160
       160
         0
```

$$16.4 \div 4.0 = 4.0\overline{)16.4} = 4.1$$

```
          4.1
4.0 / 16.4
      16
       04
        4
        0
```

EXERCISE 12:

Perform the indicated division:

1. 6/10
2. 1/6.28
3. 2.743/3.77
4. 5.372/32
5. 5.0/51
6. 0.0765/23
7. 81/0.9
8. 6/0.19
9. 48/6.254

59-20. Percentage

Percentage is the process of computation in which the basis of comparison is ONE HUNDRED. Thus, 2 percent of a quantity means two parts of every hundred parts of the quantity.

The symbol of percentage is %. Percent may also be indicated by a fraction or a decimal. Thus, $5\% = \frac{5}{100} = 0.05$.

The BASE is the number on which the percentage is computed.

The RATE is the amount (in hundredths) of the base to be estimated.

The PERCENTAGE is a part or proportion of a whole expressed as so many per hundred. Percentage is the portion of the base determined by the rate.

Conversion of Decimal to Percent:

To change a decimal to percent, move the decimal point two places to the right and add the percent symbol.

EXAMPLE:

Change 0.375 to percent

Move decimal point two places to right: 37.5

Add percent symbol: 37.5%

Conversion of Fraction to Percent:

To convert a fraction to percent, divide the numerator by the denominator and convert to a decimal. Then, convert the decimal to percent.

EXAMPLE:

Change fraction $\frac{5}{8}$ to percent.

Divide numerator by denominator: $5 \div 8 = 0.625$

Convert decimal to percent: $0.625 = 62.5\%$

Thus, $\frac{5}{8} = 62.5\%$

Conversion of Percent to Decimal:

To change a percent to a decimal, omit the percent symbol and move the decimal point two places to the left.

EXAMPLE:

Change 15% to a decimal

Omit percent symbol: 15% becomes 15

Move decimal point two places to the left: 15 becomes 0.15

Thus, 15% = 0.15

EXAMPLE:

Change 110% to a decimal.

Omit percent symbol: 110% becomes 110

Move the decimal point two places to the left: 110 becomes 1.10

Thus, 110% = 1.10

Conversion of Percent to Fraction

To change a percent to a fraction, first change the percent to a decimal and then to a fraction. Reduce the fraction to its lowest terms.

EXAMPLE:

Change 25% to a fraction

Change to a decimal: 25% = 0.25

Change to a fraction: $0.25 = \frac{25}{100}$

Reduce fraction to lowest terms: $\frac{25}{100} = \frac{1}{4}$

EXAMPLE:

Change 37.5% to a fraction

Change to a decimal: 37.5% = 0.375

Change to a fraction: $0.375 = \frac{375}{1000}$

Reduce fraction to lowest terms: $\frac{375}{1000} = \frac{3}{8}$

Finding Percentage:

To find the percent of a number, write the percent as a decimal and multiply the number by this decimal. In this case, the BASE and RATE are given. The problem is to find the percentage.

EXAMPLE:

Find 5% of 140 (140 is the base, 5% is the rate, and the product is the percentage)

5% of 140 = 0.05 x 140 = 7

XAMPLE:

Find 5.2% of 140.

5.2% of 140 = 0.052 x 140 = 7.28

EXAMPLE:

Find 150% of 36.

150% of 36 = 1.50 x 36 = 54

EXAMPLE:

Find $\frac{1}{2}$% of 840

$\frac{1}{2}\% = 0.5\%$

0.5% of 840 = 0.005 x 840 = 4.20

Thus, $\frac{1}{2}$% of 840 = 4.20

In electronics, typical applications of percentage computation are used in determining tolerance values of resistors or in determining the efficiencies of motors and generators.

nding Rate:

To find the percent one number is of another, write the problem as a fraction, change the frac-on to a decimal, and write the decimal as a percent. In this case, the PERCENTAGE and BASE are given. The problem is to find the RATE.

EXAMPLE:

3 is what percent of 8? (3 is the percentage, 8

$$\frac{3}{8} = 0.375$$

$$0.375 = 37.5\% = 37\frac{1}{2}\%$$

Therefore, 3 is $37\frac{1}{2}\%$ of 8.

EXAMPLE:

What percent of 542 is 234?

$$\frac{234}{542} = 0.4317 + \text{(round off)}$$

$$0.432 = 43.2\%$$

Therefore, 234 is 43.2% of 542.

EXAMPLE:

125 is what percent of 50?

$$\frac{125}{50} = 2.50$$

$$2.50 = 250\%$$

Therefore, 125 is 250% of 50.

Finding Base Numbers:

To find a number when a percent of the number is known, first find 1% of the number, and then find 100% of the number. In this case, the PERCENTAGE of the number and the RATE are given. The problem is to find the BASE.

EXAMPLE:

42 is 12% of what number?

12% (base number) = 42

$$1\% \text{ (base number)} = \frac{42}{12} = 3.50$$

100% (base number) = $100 \times 3.50 = 350$

The base number is 350

Therefore, 42 is 12% of 350.

EXAMPLE:

45 is 150% of what number?

150% (base number) = 45

$$1\% \text{ (base number)} = \frac{45}{150} = 0.3$$

100% (base number) = $100 \times 0.3 = 30$

The base number is 30.

Therefore, 45 is 150% of 30.

Expressing Accuracy of Measurements in Percent:

RELATIVE ERROR is the accuracy of a measurement expressed in percent of the total measurement. In determining the relative error, it is first necessary to establish the LIMIT OF ERROR.

The limit of error is the difference between the TRUE VALUE and the MEASURED VALUE. Assume that the reading on a scale, to the nearest tenth of an inch, is 2.2 inches. If the true value is 2.15 inches, the limit of error is the difference between 2.15 and 2.20, or 0.05 inch.

Relative error is computed by solving the ratio $\frac{\text{LIMIT OF ERROR}}{\text{MEASURED VALUE}}$ and expressing the result as a percent. In the scale reading above, the relative error $= \frac{0.05}{2.2} = 2.27\%$, or 2.3%.

EXERCISE 13:

Show each of the following in three forms—as a fraction or mixed number, as a decimal, and as a percent:

1. $\frac{3}{5}$
2. 50%
3. 0.375
4. $\frac{1}{4}$
5. $62\frac{1}{2}\%$
6. 0.6
7. $\frac{3}{10}$
8. 70%
9. 2.25
10. $1\frac{7}{8}$
11. 0.08
12. $\frac{3}{50}$
13. 0.18
14. $\frac{1}{4}\%$
15. 0.025
16. 0.05
17. $8\frac{1}{3}\%$
18. $37\frac{1}{2}\%$
19. 105%
20. 4%

Evaluate the following:

21. 250% of 60
22. 125% of 40
23. 200% of 2
24. 225% of 400

What percent of a number is:

25. 1.5 times the number?
26. $2\frac{3}{4}$ times the number?
27. $\frac{3}{2}$ times the number?
28. $5\frac{1}{2}$ times the number?

Find the following:

29. $\frac{2}{5}\%$ of 410
30. $\frac{3}{5}\%$ of 416,000
31. $\frac{2}{5}\%$ of 85
32. 5.2% of 85

Solve the following problems:

33. Find the relative error for a limit of error of 0.05 inch in measuring 24.2 inches.
34. Find the relative error for a limit of error of 2 inches in measuring 200 yards.

Find the number when:

35. 12% of the number is 52
36. 15% of the number is 375
37. 32% of the number is 166.4
38. 8% of the number is 16
39. 84% of the number is 168
40. 17% of the number is 22

1-21. Equations and Transposition

An EQUATION is a statement of quality between two expressions. For example, $x + y = 12$, $3x + 5 = 20$, and $3 \times 9 = 27$ are equations; therefore, all expressions separated by the equality are equations, whether the expressions are algebraic or arithmetical. The expression to the left of the equality sign is called the LEFT HAND MEMBER of the equation; the expression to the right of the equality sign is called the RIGHT HAND MEMBER. Finding the values of the unknown quantities of an algebraic equation is known as solving the equation, and the answer is called the SOLUTION. If only one unknown is involved, the solution is also called the ROOT.

Solving simple equations:

1. Equal quantities may be added to both sides of an equation without changing the quality.

EXAMPLE:

Solve the equation $x - 4 = 7$ for x

$$x - 4 = 7$$

$$x - 4 + 4 = 7 + 4$$

$$x = 11$$

Solve the equation $x - 7 = 14$ for x

$$x - 7 = 14$$

$$x - 7 + 7 = 14 + 7$$

$$x = 21$$

2. Equal quantities may be subtracted from both sides of an equation without changing the quality.

EXAMPLE:

Solve the equation $x + 2 = 5$ for x

$$x + 2 = 5$$

$$x + 2 - 2 = 5 - 2$$

$$x = 3$$

Solve the equation $x + 5 = 12$ for x

$$x + 5 = 12$$

$$x + 5 - 5 = 12 - 5$$

$$x = 7$$

3. Both sides of an equation may be multiplied by the same number without changing the equality.

EXAMPLE:

Solve the equation $\frac{x}{3} = 5$ for x

$$\frac{x}{3} = 5$$

$$\frac{x}{\cancel{3}} \times \frac{\cancel{3}}{1} = 5 \times 3$$

$$x = 15$$

Solve the equation $\frac{x}{3} + \frac{x}{9} = 4$ for x

Multiply both sides of the equation by 9.

$$\frac{x}{\cancel{3}} \times \frac{\overset{3}{\cancel{9}}}{1} + \frac{x}{\cancel{9}} \times \frac{\cancel{9}}{1} = 4 \times 9$$

$$3x + x = 36$$

$$4x = 36$$

4. Both sides of an equation may be divided by the same quantity without changing the quality.

EXAMPLE:

Solve the equation $3x = 12$ for x

$$3x = 12$$

$$\frac{\cancel{3}x}{3} = \frac{\overset{4}{\cancel{12}}}{\cancel{3}}$$

$$x = 4$$

Solve the equation $PV = RT$ for T

$$PV = RT$$

$$\frac{PV}{R} = \frac{\cancel{R}T}{\cancel{R}}$$

$$T = \frac{PV}{R}$$

Solving more difficult equations:

The process of adding to or subtracting from both members of an equation can be shortened by shifting a term or terms from one side of the equation to the other and changing the signs. This operation is called TRANSPOSITION.

EXAMPLE:

Solve the equation $6x + 4 = x - 16$ for x

$$6x + 4 = x - 16$$

$$6x - x = -16 - 4$$

$$5x = -20$$

$$x = -4$$

Solve the equation $5a - 7 = 2a + 2$ for a

$$5a - 7 = 2a + 2$$

$$5a - 2a = 2 + 7$$

$$3a = 9$$

$$a = 3$$

In solving a fractional equation, first find the LCD and multiply both members of the equation, term by term; then perform the operations described previously.

EXAMPLE:

Solve the equation $\frac{x}{2} + \frac{x}{3} = 10$ for x

$$\frac{x}{2} + \frac{x}{3} = 10$$

$$\frac{3x + 2x}{6} = 10$$

$$\frac{5x}{6} = \frac{10}{1}$$

$$5x = 60$$

$$x = 12$$

Written equations:

Many practical problems are stated in words must be translated into symbols before the rules of algebra can be applied. There are no specific rules for the translation of a written problem into an equation of numbers, signs and symbols. The following general suggestions may be helpful in developing equations:

a. From the worded statement of the problem, select the unknown quantity (or one of the unknown quantities) and represent it by a letter such as X. Write the expression, stating exactly what X represents and the units in which it is measured.

b. If there is more than one unknown quantity in the problem, try to represent each unknown in terms of the first unknown.

EXAMPLE:

In simple problems, an equation may be written by an almost direct translation into algebraic symbols; thus,

Seven times a certain number diminished by 3 gives the same result as the number increased by 75

$$7 \quad \times \quad Z \quad - \quad 3 \quad = \quad Z \quad + \quad 75$$

Solving the equation:

$$7Z - 3 = Z + 75$$

$$7Z - Z = 75 + 3$$

$$6Z = 78$$

$$Z = 13$$

Check:

$$7(13) - 3 = 13 + 75$$

$$91 - 3 = 13 + 75$$

$$88 = 88$$

A triangle has a perimeter of 30 inches. The longest side is 7 inches longer than the shortest side and the third side is 5 inches longer than the shortest side. Find the length of the three sides.

Let x = length of the shortest side

$x + 7$ = length of the longest side

$x + 5$ = length of the third side

$$x + (x + 7) + (x + 5) = 30$$

Solving the equation:

$$x + x + 7 + x + 5 = 30$$

$$3x + 12 = 30$$

$$3x = 30 - 12$$

$$3x = 18$$

$x = 6$ - shortest side

$6 + 7 = 13$ longest side

$6 + 5 = 11$ third side

Simultaneous equations:

Simultaneous equations are two or more equations satisfied by the same sets of values of the unknown quantities. Simultaneous equations are used to solve a problem containing two or more unknown quantities. A general rule for establishing a set of simultaneous equations is that for every unknown quantity in the problem there must be an equation in the set of simultaneous equations. Thus, for two unknowns in the problem there will be two equations, three unknowns, three equations, etc.

In the solution of simultaneous linear equations three methods will be explained by the use of an example. The methods which will be used are addition, subtraction and substitution.

EXAMPLE:

Assume that the sum of two numbers is 17, and that three times the first number less two times the second number is equal to 6. What are the numbers? In setting up equations for this problem, let x equal the first number and y equal the second number. The first equation is $x + y = 17$, and the second equation is $3x - 2y = 6$.

1. Addition

$$x + y = 17$$

$$3x - 2y = 6$$

Before adding, change the y in the first equation to 2y so that the y terms drop out when added, thus, the first equation must be multiplied by 2.

$$\begin{array}{r} 2x + 2y = 34 \\ 3x - 2y = 6 \\ \hline 5x = 40 \\ x = 8 \end{array}$$

Substitute $x = 8$ in the first equation and solve for y:

$$x + y = 17 \text{ or } 8 + y = 17$$

$$y = 17 - 8$$

$$y = 9$$

2. Subtraction

$$x + y = 17$$

$$3x - 2y = 6$$

Before subtracting, multiply the first equation by 3 so that the x terms drop out when subtracted.

$$\begin{array}{r} 3x + 3y = 51 \\ 3x - 2y = 6 \\ \hline 5y = 45 \\ y = 9 \end{array}$$

Substitute $y = 9$ in the first equation and solve for y:

$$x + y = 17 \text{ or } x + 9 = 17$$

$$x = 17 - 9$$

$$x = 8$$

3. Substitution

$$x + y = 17 \text{ or } x = 17 - y$$

Substitute $x = 17 - y$ in the second equation:

$$3x - 2y = 6$$

$$3(17 - y) - 2y = 6$$

Remove the parenthesis:

$$51 - 3y - 2y = 6$$

Transpose:

$$-5y = 6 - 51$$

$$-5y = -45$$

$$5y = 45$$

$$y = 9$$

Substitute $y = 9$ in the first equation and solve for x:

$$x + y = 17 \text{ or } x + 9 = 17$$

$$x = 17 - 9$$

$$x = 8$$

Solving Formulas:

A formula is a rule or law that states a scientific relationship. It can be expressed in an equation by using letters, symbols and constant terms.

To solve a formula, perform the same operations on both members of an equation until the desired unknown can be isolated in one member of the equation. If the numerical values for some variables are given, substitute in the formula and solve for the unknown as in any other equation.

EXAMPLE:

1. Solve the formula $T = \frac{12\,(D-d)}{t}$ for D.

$$T = \frac{12\,(D-d)}{t}$$

$$T = \frac{12D - 12d}{t}$$

Multiply both sides by t:

$$Tt = 12D - 12d$$

Transpose and change signs:

$$12D = Tt + 12d$$

Divide both sides by 12:

$$\frac{12D}{12} = \frac{Tt}{12} + \frac{12d}{12}$$

$$D = \frac{Tt}{12} + d$$

2. Given the formula $R_T = \frac{R_1\,R_2}{R_1 + R_2}$ solve for R_2.

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Cross multiply:

$$R_1 R_2 = R_1 R_T + R_2 R_T$$

Transpose and change signs:

$$R_1 R_2 - R_2 R_T = R_1 R_T$$

Factor R_2 out of the left member:

$$R_2\,(R_1 - R_T) = R_1 R_T$$

Divide both sides by $R_1 - R_T$

$$\frac{R_2 (R_1 - R_T)}{R_1 - R_T} = \frac{R_1 R_T}{R_1 - R_T}$$

$$R_2 = \frac{R_1 R_T}{R_1 - R_T}$$

59-22. Square Roots

A FACTOR of a number is an even division of that number. As an example, the numbers 3 and 5 are even divisors of the number 15. Therefore, both 3 and 5 are factors of the number 15. In the number 4, the number 2 is a factor twice ($2 \times 2 = 4$). The numbers in the latter example are called EQUAL FACTORS. When a number contains only equal factors, they are called ROOTS of the number. When a number is divided into two equal factors, either of the equal factors are called SQUARE ROOT of the number.

The sign used to indicate that a square root is to be extracted is $\sqrt{}$. It is placed over the number whose square root is to be found. As an example, the square root of sixteen ($\sqrt{16}$) would mean that two equal factors of the number are to be found. The sign indicating the extraction ($\sqrt{}$) is called a RADICAL sign.

The following method is used to extract the square root of the number 401,956.

Step One:

Begin at the decimal point (which is to the right of the last digit) and divide the number into two digit groups in both directions.

$$\sqrt{401,956}$$

Step Two:

Place the decimal point for the square root directly above the decimal point that appears under the radical sign.

$$\begin{array}{r} . \\ \sqrt{40\ 19\ 56.} \end{array}$$

Step Three:

Determine the largest number that, when multiplied by itself, will give a product equal to or less than the first pair of digits 40. The number 6, since any number larger than 6 multiplied by itself will produce a number greater than 40. Place the 6 above the first pair of digits.

$$\begin{array}{l} \ \ 6 \qquad . \\ \sqrt{40\ 19\ 56.} \end{array}$$

Step Four:

Square 6 to obtain 36 and place it below the first two digits 40. Subtract 36 from 40 and obtain 4. Bring down the next pair of digits 19.

$$\begin{array}{l} \ \ 6 \qquad . \\ \sqrt{40\ 19\ 56.} \\ \ \ \underline{36} \\ \ \ \ \ 4\ 19 \end{array}$$

Step Five:

Double the first digit of the answer, 6, to obtain a TRIAL DIVISOR of 12. Place the 12 to the left of 419 as shown.

$$\begin{array}{ll} & \ \ 6 \qquad . \\ & \sqrt{40\ 19\ 56.} \\ & \ \ \underline{36} \\ 12 & \ \ \ \ 4\ 19 \end{array}$$

Step Six:

Divide the trial divisor (12) into all but the last digit of the modified remainder — 419. It will divide into 41 three times. This will be the next digit of the answer. Place the three above the second pair of digits and also place the 3 to the right of the trial divisor. Thus, the completed divisor is 123. Multiply 123 by 3 and obtain 369. Subtract 369 from 419 to obtain 50. Bring down the next pair of digits 56.

```
              6  3  .
            √40 19 56.
             36
     123      4 19
              3 69
                50 56
```

Step Seven:

Double the first two digits of the answer 63, to obtain the new trial divisor of 126. Place the 126 to the left of 5056 as shown.

```
              6  3  4.
            √40 19 56.
             36
     123      4 19
              3 69
     126        50 56.
```

Step Eight:

Divide the trial divisor 126 into all but the last digit of the modified remainder 5056. It will go into 505 four times. This will be the next digit of the answer. Place the four above the third digit, and also place the 4 to the right of the trial divisor. Thus, the completed divisor is 1264, multiply 1,264 by 4 and obtain 5056. Subtract 5056 from 5056. The remainder is zero. Therefore, the square root of 401,956 is 634.

Step Nine:

Check the answer by squaring 634.

634 x 634 = 401,956

EXAMPLE:

Find the square root of 552.35

Step One:

Begin at the decimal point and divide the number into digit groups in both directions.

```
√05 52. 35
```

Step Two:

Place the decimal point for the square root directly above the decimal point that appears under the radical sign.

```
        .
√05 52. 35
```

Step Three:

Determine the largest number that when multiplied by itself will give a product equal to or less than the first pair of digits, 05. The number is 2 since any number larger than 2 multiplied by itself will produce a number greater than 5. Place the 2 above the first pair of digits.

```
   2    .
√05 52. 35
```

Step Four:

Square 2 to obtain 4 and place it below the first two digits, 05. Subtract 4 from 5 and obtain 1. Bring down the next pair of digits, 52.

```
   2    .
√05 52. 35
   4
   1 52
```

Step Five:

Double the first digit of the answer 2, to obtain a trial divisor of 4. Place the four to the left of 152 as shown.

```
             2  .
          √05 52. 35
            4
4           1 52
```

Step Six:

Divide the trial divisor (4) into all but the last digit of the modified remainder 152. It will divide into 15 three times. This will be the next digit of the answer. Place the 3 above the second pair of digits and also place the three to the right of the trial divisor. Thus, the completed divisor is 43. Multiply 43 by 3 and obtain 129. Subtract 129 from 152 to obtain 23. Bring down the next pair of digits, 35.

```
             2  3.
          √05 52. 35
            4
43          1 52
            1 29
              23  35
```

Step Seven:

Double the first two digits of the answer, 23, to obtain the new trial divisor of 46. Place the 46 to the left of 2335 as shown.

```
             2  3.
          √05 52. 35
            4
43          1 52
            1 29
46            23  35
```

Eight:

Divide the trial divisor 46 into all but the last digit of the modified remainder, 2335. It will divide into 233 five times. This will be the next digit of the answer. Place the 5 above the pair of digits and to the right of the trial divisor. Thus, the completed divisor is 465. Multiply 465 by 5 and obtain 2325. The remainder is 10. Therefore, the square root of 552.35 is 23.5 with a remainder of 10. A more accurate answer can be obtained by adding zeros and continuing the process of extracting the square root.

```
             2  3.  5
          √05 52. 35
            4
43          1 52
            1 29
465           23  35
              23  25
                  10
```

Step Nine:

Check the answer by multiplying 23.5 by itself and adding the remainder.

$$23.5 \times 23.5 = 552.25$$

$$552.25 + 0.10 = 552.35$$

Therefore, the square root of 552.35 equals 23.5 with a remainder of 10.

Square Root of a Product:

If the factors of the product are perfect squares, the square root of the product is obtained by ex- ting the square root of the individual factors and multiplying them together.

AMPLE:

$$\sqrt{64 \times 9} = \sqrt{64} \times \sqrt{9} = 8 \times 3 = 24$$

EXAMPLE:

$$\sqrt{16 \times 144} = \sqrt{16} \times \sqrt{144} = 4 \times 12 = 48$$

EXAMPLE:

$$\sqrt{169 \times 25} = \sqrt{169} \times \sqrt{25} = 13 \times 5 = 65$$

If the factors are not each a perfect square, find the product of the factors and extract the square root.

EXAMPLE:

$$\sqrt{6 \times 8} = \sqrt{48} = 6.93$$

The square root of a fraction is equal to the square root of the numerator divided by the square root of the denominator.

EXAMPLE:

$$\sqrt{25/169} = \frac{\sqrt{25}}{\sqrt{169}} = 5/13$$

EXAMPLE:

$$\sqrt{16/81} = \frac{\sqrt{16}}{\sqrt{81}} = 4/9$$

If the fractions numerator and denominator are not perfect squares, it would be easier to reduce the fraction to its decimal form and extract the square root.

EXAMPLE:

$$\sqrt{2/6} = \sqrt{0.333} = 0.58$$

EXERCISE 14:

Extract the square root of the following:

1. 3,969 2. 1,049.76 3. 275.56 4. 49,729

Extract the square root to two decimal places.

11. 684 12. 321 13. 2 14. 5

15. 86 16. $\sqrt{16/64}$ 17. $\sqrt{3/8}$ 18. $\sqrt{64 \times 16}$

RATIO AND PROPORTION

59-23. Ratio

A knowledge of "how many" of a certain group of objects or quantities may have little meaning in a discussion, unless that quantity is compared to another quantity. For example, to say that a man has the ability to read 400 words in one minute has little meaning without comparing his rate to another. However, when his rate is compared to the 250 words per minute rate of the average reader, one can see that he is capable of a considerably higher reading rate than the average reader. To determine this comparison mathematically, his rate is divided by the average.

EXAMPLE:

$$\frac{400 \text{ words per minute}}{250 \text{ words per minute}} = \frac{400}{250} = \frac{8}{5} = 1\frac{3}{5}$$

Thus, for every 500 words read by the average reader, this man reads 8/5 or $1\frac{3}{5}$ as fast.

It is only through such comparisons that many numbers have meaning. When a relationship between two numbers is shown in this manner, they are compared as a RATIO. A ratio is a comparison of two like quantities. It is the quotient obtained by dividing the first number by the second.

For example, a gear has 40 teeth and another gear has 10 teeth. A comparison would be 40 teeth to ten teeth. This comparison could be written as a ratio in four ways: 40:10, 40 ÷ 10, 40/10, of the ratio of 40 to 10. When the emphasis is on ratio, all of these expressions would be read, "the ratio of 40 to 10".

Two quantities expressed as a ratio must be of the same kind. For example, there can be no single ratio between 12 bolts and five men. A ratio should be expressed in similar units, yards to yards, quarts to quarts, etc.

The two numbers of a ratio are called the TERMS of the ratio. The first term, or the numerator, s called the ANTECEDENT. The second term, or the denominator, is called the CONSEQUENT. In e previous example, the number 40, is the antecedent, and the number 10 is the consequent.

Since a ratio is also a fraction, all of the rules governing the operation of fractions apply to operations with ratios. Therefore, the ratios may be reduced, simplified, increased, decreased, and so forth. To reduce the terms of a ratio, such as 15 to 20, write the ratio as a fraction, and proceed as in fractions. Thus, 15 to 20 becomes:

$$\frac{15}{20} = \frac{3 \times 5}{4 \times 5} = \frac{3}{4} \cdot \frac{5}{5}$$

Since the fraction $\frac{5}{5} = 1$

Then $\frac{3}{4} \times \frac{5}{5} = \frac{3}{4}$

Therefore: $\frac{15}{20} = \frac{3}{4}$

Hence, the ratio of 15/20 is the same as the ratio 3/4.

Notice the distinction in thought between 3/4 as a fraction and 3/4 as a ratio. As a fraction, 3/4 is the single quantity "three-fourths". As a ratio, 3/4 is a comparison between two numbers, 3 and 4.

EXAMPLE:

The length of two sides of a triangle are 6 ft and 2 ft. To compare their lengths by means of a ratio, divide one number by the other and reduce to lowest terms.

$$\frac{6 \text{ ft}}{2 \text{ ft}} = \frac{3 \times 2 \times \text{ft}}{1 \times 2 \times \text{ft}} = \frac{3}{1} \times \frac{2}{2} \times \frac{\text{ft}}{\text{ft}} = \frac{3}{1} \times 1 \times 1 = \frac{3}{1}$$

The two sides of the triangle compare as 3 to 1.

59-24. Proportion

Closely allied with the study of ratio is the subject of PROPORTION. A proportion is nothing more than an equation in which the members are ratios. In other words, when two ratios are set equal to one another, a proportion is formed. When any quantity is set equal to another quantity, an equation is formed. The proportion may be written in three different ways. The fact that 15 to 20 is the same as 3 to 4 may be expressed as:

$$15:20 :: 3:4$$

$$15:20 = 3:4$$

$$\frac{15}{20} = \frac{3}{4}$$

The last two forms in this example are the most common. They may be read as "15 is to 20 as 3 is to 4". In other words, 15 has the same ratio of 20 as 3 has to 4.

One reason proportions are so important is that if any of three of the terms are given, the fourth may be found by a simple equation. In science, many chemical and physical relationships are expressed as proportions. Consequently, a familiarity with proportions will provide one method of solving applied problems. It is evident from the last form shown, 15/20 equals 3/4, that a proportion is really a fractional equation, and as such all rules for fractional equations apply.

Certain names have been given to the terms of the two ratios that make up a proportion. In a proportion the first and the last terms are called the EXTREMES. In other words the antecedent of the rst ratio and the consequent of the second are called the extremes. The second and third terms are called the MEANS. The means are the consequent of the first ratio and the antecedent of the second. Summarizing,

$$5:4 = 15:12$$

means

It is often advantageous to change the form of a proportion. There are several rules for changing or combining the terms of a proportion without altering the equality between the members. These rules are simplifications of proven fundamental rules for arithmetical and algebraic equations. Emphasis is placed upon these rules because they are used constantly and it is in the students best interest to memorize them.

RULE No. 1. In any proportion the product of the means is equal to the product of the extremes.

EXAMPLE:

$$\frac{2}{3} = \frac{6}{9} \quad \text{therefore } 3 \times 6 = 2 \times 9$$

EXAMPLE:

$$\frac{a}{b} = \frac{c}{d} \quad \text{therefore } bc = ad$$

RULE No. 2. The product of the means divided by either extreme gives the other extreme.

EXAMPLE:

$$\frac{2}{3} = \frac{6}{9} \; ; \; \frac{3 \times 6}{9} = 2 \text{ or } \frac{3 \times 6}{2} = 9$$

EXAMPLE:

$$\frac{a}{b} = \frac{c}{d} \; ; \; \frac{bc}{d} = a \text{ or } \frac{bc}{a} = d$$

RULE No. 3. The product of the extremes divided by either mean gives the other mean.

EXAMPLE:

$$\frac{2}{3} = \frac{6}{9} \; ; \; \frac{2 \times 9}{3} = 6 \text{ or } \frac{2 \times 9}{6} = 3$$

EXAMPLE:

$$\frac{a}{b} = \frac{c}{d} \; ; \; \frac{ad}{b} = c \text{ or } \frac{ad}{c} = b$$

Solving for the Unknown Term:

Using the rules of proportion solve the below proportions for the unknown term.

EXAMPLE:

$$\frac{3}{4} = \frac{9}{Y}$$

Rule 2 says $$\frac{4 \times 9}{3} = Y$$

Therefore, $$Y = \frac{36}{3} = 12; \qquad \frac{3}{4} = \frac{9}{12}$$

EXAMPLE:

$$\frac{Y}{6} = \frac{12}{18}$$

Rule 3 says $$\frac{6 \times 12}{18} = Y$$

Therefore: $72 =$ … $4 = 12$

EXERCISE 15:

Work the problems below and check your answers.

1. $\frac{6}{x} = \frac{18}{3}$ 2. $\frac{3}{4} = \frac{x}{8}$ 3. $\frac{x}{5} = \frac{8}{20}$

4. $\frac{6 \text{ ft}}{3 \text{ ft}} = \frac{x}{4 \text{ min}}$ 5. $\frac{60 \text{ miles}}{120 \text{ miles}} = \frac{180 \text{ min}}{x}$

A knowledge of the properties of a proportion often provides a quick and easy method of solving word problems. However, when setting up proportion problems, be sure that the ratios are stated correctly. The ratios must be compared in the same order. In other words, if one ratio is compared lesser to greater then the other ratio must be compared in the same order. Therefore, you must analyze the problem and decide whether the unknown quantity will be lesser or greater than the known quantity of the ratio in which it occurs. The following examples will show the processes involved in setting up and solving ratio and proportion problems.

EXAMPLE:

If an automobile runs 60 miles on 6 gallons of gas, how many miles will it run on 20 gallons?

$$\frac{\text{LESSER}}{\text{GREATER}} = \frac{\text{LESSER}}{\text{GREATER}}$$

It is known that 6 gallons of gasoline is less than 20 gallons. If the automobile travels 60 miles on the 6 gallons of gasoline, how far will it travel on 20 gallons?

Let Z equal the unknown.

$$\left(\frac{\text{Lesser}}{\text{Greater}}\right) \frac{60 \text{ miles}}{Z} = \frac{6}{20} \left(\frac{\text{Lesser}}{\text{Greater}}\right)$$

Using Rule 2:

$$Z = \frac{20 \times 60 \text{ miles}}{6}$$

$$Z = \frac{20 \times 10 \text{ miles}}{1}$$

$$Z = 20 \times 10 \text{ miles}$$

$$Z = 200 \text{ miles}$$

EXAMPLE:

A pulley 60 inches in diameter is turning at a speed of 600 revolutions per minute. This pulley is connected to another pulley with a diameter of 30 inches. What is the revolutions per minute of the smaller pulley?

$$\frac{\text{LESSER}}{\text{GREATER}} = \frac{\text{LESSER}}{\text{GREATER}}$$

Ratios must be expressed between quantities of the same kind.

Let the RPM of the smaller pulley be represented by X. Analyze the problem: one ratio will be between inches and the other between revolution per minute (RPM). Also note that the smaller pulley ill make more revolutions per minute than the larger one. Therefore, the answer will have to be rger than 300. Arrange the ratios in the order lesser to greater:

Ratio of Inches $\frac{30 \text{ inches}}{60 \text{ inches}} = \frac{30}{60}$

Ratio of RPM $\frac{600 \text{ RPM}}{X} = ?$

The proportion:

$$\frac{30}{60} \quad \frac{600\ RPM}{X}$$

extreme — 30; mean — 60; mean — 600 RPM; extreme — X

Using the rule which states that the product of the means divided by either extremes equals the other extreme.

$$\frac{\underset{\text{mean}}{60} \times \underset{\text{mean}}{600\ RPM}}{\underset{\text{extreme}}{30}} = \underset{\text{extreme}}{X}$$

$$\frac{2 \times 1 \times 600\ RPM}{1} = X$$

$$2 \times 600\ RPM = X$$

$$1200\ RPM = X$$

The proportion above is called an INVERSE proportion, because the smaller the diameter of the pulley the faster it will rotate.

Two numbers are inversely proportional when an increase in one causes the other to decrease, or a decrease in one causes an increase in the other.

EXERCISE 16:

In each of the following problems, set up the correct proportions and solve for the unknown value.

1. Find the fourth proportional of 6, 3, and 12 (taken in order).
2. If a mast 8 ft high casts a shadow 10 ft long, how high is a mast that casts a shadow 40 ft long?
3. The speed of two cars is in a ratio of 2 to 5. If the slower car goes 30 mph, what is the speed of the faster car?
4. If 6 seamen can empty 2 cargo spaces in a day, how many spaces can 150 seamen empty in a day?
5. If 12 typewriters cost $1,020, how much will 9 cost at the same rate?

ALGEBRA

ALGEBRA may be thought of as an extension of arithmetic because it extends the concept of numbers. In algebra, numbers can be represented by letters of the alphabet. The letters are called LITERAL NUMBERS. Literal numbers are used to express known or unknown quantities. Literal numbers are also used to show relationships between quantities which are related through a physical law. For example, the algebraic expression $I = E/R$ shows that (I) will vary if (E) or (R) is varied. The relationship between I, E, and R is constant, but the numerical values for these letters may take on many different values.

Before attempting the use of algebra as a tool, knowledge of basic operations and definitions must be acquired. Some basic operations and definitions are listed.

59-25. Definitions and Rules

The following signs, used in algebra, have the same meaning which they have in arithmetic. These signs are +, -, ÷, and X. They indicate addition, subtraction, division and multiplication respectively.

Order of Operations:

When there are multiplications, divisions, additions and subtractions to be performed on a group of numbers, multiplication must be performed first, division second, and then the addition and subtraction.

EXAMPLE:

$$32 \div 4 + 3 + 2 \times 4 - 6 = X$$

$$8 + 3 + 8 - 6 = X$$

$$13 = X$$

Expression:

An algebraic EXPRESSION is a group of letters and numbers.

Factor:

Whenever two or more numbers are multiplied together, they are called FACTORS. In the expression $2\pi fL$, two is a factor of $2\pi fL$. Also $2\pi f$ is a factor of $2\pi fL$.

Terms:

A TERM of an algebraic expression is the parts of the expression not separated by a plus or minus sign. In the expression $5X+6Y-3Z$, the terms are 5X, 6Y, and 3Z.

If an algebraic expression has one term it is a MONOMIAL. An expression containing two terms is called a BINOMIAL. A TRINOMIAL has three terms. An expression containing more than three terms is called a POLYNOMIAL.

Subscript:

A SUBSCRIPT is a number or letter written to the right and the bottom of another number for further identification. In the equation $X_L = 2\pi fL$. The subscript, L is next to the X. The equation is read "X sub L is equal to two pi f L".

Laws of Exponents:

An EXPONENT in a number placed to the right and above another number (the BASE) to indicate the number of times the base is to be taken as a factor.

When like bases are multiplied together, add the exponents. When like bases are divided, subtract the exponent in the divisor from the exponent in the dividend. Examples of these operations are:

$$a^2 \cdot a^3 = a^{2+3} = a^5$$

$$a^3 / a^2 = a^{3-2} = a$$

$$a^4 / a^{-2} = a^{4+2} = a^6$$

Square Root:

To extract the SQUARE ROOT of an algebraic expression, find the two equal factors of the expression. Both factors are square roots. Examples of this operation are:

$$\sqrt{a^4b^8} = a^2b^4$$

$$\sqrt{a^8b^4x^2} = a^4b^2x$$

Negative Numbers:

NEGATIVE NUMBERS can be considered to be numbers with a value less than zero. They are necessary if subtractions like 4-8 are to be performed. Negative numbers have a physical meaning when applied to the appropriate quantities. It is absurd to speak of a physical length which is less than zero units long, but it is quite common to speak of a temperature that is less than zero degrees. The idea of applying the appropriate numbers to physical quantities should be considered whenever working with numbers.

Real Numbers:

The REAL number system contains both POSITIVE and NEGATIVE numbers. Figure 59-1 shows the graphical representation of the real number system. On a NUMBER SCALE positive numbers are plotted to the right of zero. Negative numbers are plotted to the left of zero. The zero point is frequently called the ORIGIN.

Addition:

The two basic rules for addition are:

1. To add two or more numbers with like signs, add the magnitudes and prefix the common sign.

2. To add a positive and negative number, compute the difference between the magnitudes and prefix the sign of the larger magnitude.

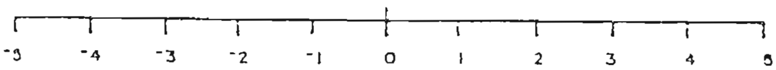


Figure 59-1 - Real numbers on the number scale.

Examples of this operation are:

30	-40	-40
40	20	20
70	-20	-30
		-50

EXERCISE 17:

Add the following:

1.	2.	3.	4.	5.
80	-250	-60	102	-86
-30	50	-40	98	96
60	180	-200	-60	-11

Subtraction:

The rule for subtraction is:

1. To subtract, change the sign of the subtrahend and add.

Examples of this operation are:

40	-100	-250	300
-60	60	-75	400
100	-160	-175	-100

EXERCISE 18:

Perform the subtraction of the following:

1.	2.	3.	4.	5.
-275	-415	660	450	722
-113	115	320	620	-131

Multiplication of Real Numbers:

The rules for multiplication are:

1. The product of two numbers that have like signs is a positive number.

2. The product of two numbers that have unlike signs is a negative number.

Examples of this operation are:

$45 \times 30 = 1350$ $(-20) \times (-10) = 200$ $(-405) \times (2) = -810$

EXERCISE 19:

Perform the following:

1. $(-420) \times (-30) =$
2. $(60) \times (-30) =$
3. $(-200)(40) =$
4. $(-31)(62) =$
5. $(811)(-22) =$

Division of Real Numbers:

The rules for the division of real numbers are:

1. The quotient of the division of like terms is positive.
2. The quotient of the division of unlike terms is negative.

Examples of this operation are:

$50/(-10) = -5$ $\quad$ $650/25 = 26$ $\quad$ $(-40)/(-4) = 10$

EXERCISE 20:

Perform the following operations:

1. $100/(-25) =$
2. $(-60)/(-4) =$
3. $600/(-30) =$
4. $891/(-62) =$
5. $322/(-12) =$

Grouping Terms:

If the additions, subtractions, multiplications and divisions are performed in a single operation; the terms of the expression are frequently grouped. GROUPING clarifies the operations. Terms are grouped using BRACKETS, [] ; BRACES, { }; PARENTHESIS ( ).

Examples of this operation are:

$$2\,[\{5 \times (6-3)\} - (18/9)]$$

The operations inside the inner most signs of grouping are performed first. The evaluation of the above expression is:

$$2\,[\{5 \cdot 3\} - 2] = 26$$

In removing the signs of grouping from an expression, the sign of each term within the grouping should be changed if the grouping symbol is preceded by a negative sign. The signs of the terms should not be changed if the grouping sign is preceded by a positive sign.

An example of this operation is:

$$50 - (30 + 10) + (20 - 5) - (8 \times 2)$$

To evaluate this expression, the parenthesis are removed. The expression then becomes:

$$50 - 30 - 10 + 20 - 5 - 16 = 9$$

EXERCISE 21:

Perform the following operations:

1. $5 \times (4-3) + (6-2) - (-10 + 5) =$
2. $-(4-3) + (6+10) =$
3. $-[5 + 6 - (3+2)] =$
4. $10\,[20 + 30 - (5 + 10)] =$
5. $-6\,[3 + 21 - (8 + 11) + (16 + 2) - (3 - 21)] =$

Monomials:

A MONOMIAL is an expression consisting of one term. The expressions 5a, -6b, 3X, and $4X^2$ are all classified as monomials.

Addition of Monomials:

Monomials may be added or subtracted if they are like quantities. The quantity 5a cannot be directly added to 6b unless a is similar in nature to b. If "a" indicates resistors and "b" indicates inductors, it would be meaningless to talk of the sum of five resistors and six inductors. If "a" and "b" represent other numbers then the addition is easily carried out. The addition of 5a and 6b is written in this manner.

$$5a + 6b$$

Monomials made up of like terms can be added directly. Consider the addition of 2a and 4a.

$$2a + 4a = 6a$$

In this operation, the numerical values were added directly. The numerical coefficients tell "how much", and the literal factors tell "of what". The addition of 2a and 4a can be represented by:

$$(a+a) + (a+a+a+a) = a+a+a+a+a+a$$

$$2a \quad + \quad 4a \quad = 6a$$

Subtraction of Monomials:

To subtract a monomial A from a monomial B means to find a monomial C such that C added to A gives B.

$$6a - 4a = 2a$$

In the above equation, 4a is subtracted from 6a. To perform this subtraction a monomial was found which when added to 4a gave 6a.

Examples of this operation are:

$$12a - 14a = -2a \qquad\qquad -10b - 6b = -16b$$

Multiplication of Monomials:

When monomials are multiplied, the coefficients (numerical values) are multiplied algebraically. Literal numbers are multiplied using the laws of exponents.

Examples of this operation are:

$$15a \cdot 4a = 60a^2 \qquad\qquad -5a \cdot 3a = -15a^2$$

$$(-6b)(-3b) = 18b^2 \qquad\qquad 4a(-3b) = -12ab$$

Division of Monomials:

In the division of monomials, the rule of exponents is applied to the literal factors. The numerical coefficients are divided the same as the other real numbers. Literal numbers are divided using the laws of exponents.

Examples of this operation are:

$$16Z/4Z = 4 \qquad\qquad 18W^2/6W = 3W$$

$$-15Y^2/3Y = -5Y \qquad\qquad -40X^2/-10X = 4X$$

Square Root:

Some basic rules to be observed in extracting the square root of a number are:

1. The square root of a product is equal to the product of the square root.

Examples of this operation are:

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{200} = \sqrt{2 \cdot 100} = \sqrt{2} \times \sqrt{100} = \sqrt{2} \times 10 = 14.14$$

$$\sqrt{4\pi^2 LC} = \sqrt{4\pi^2} \cdot \sqrt{LC} = 2\pi \cdot \sqrt{LC}$$

2. The square roots of a ratio (division) is equal to the ratio of the square roots.

Examples of this operation are:

$$\sqrt{a/b} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\sqrt{4/9} = \frac{\sqrt{4}}{\sqrt{9}} = 2/3$$

3. The square root of a sum is equal to the square root of that sum.

Examples of this operation are:

$$\sqrt{a + b} = \sqrt{a + b}$$

$$\sqrt{10 + 15} = \sqrt{25} = 5$$

The square root of ten plus fifteen is not equal to the square root of ten plus the square root of fifteen.

4. The square root of an indicated subtraction is equal to the square roots of that subtraction.

Examples of this operation are:

$$\sqrt{a - b} = \sqrt{a - b}$$

$$\sqrt{110 - 10} = \sqrt{100} = 10$$

The square root of one hundred and ten is not equal to the square root of one hundred minus the square root of ten.

59-26. Scientific Notation (Powers of Ten)

The technique of using powers of 10 can greatly simplify mathematical calculations. A number containing many zeros to the right or to the left of the decimal point can be delt with much more readily when put in the form of powers of 10. For example 0.0000037 x 0.000021 can be handled more easily when put in the form $3.7 \times 10^{-6} \times 2.1 \times 10^{-5}$.

Table of Powers of 10: The table below gives some of the values of the powers of 10.

Number	Powers of 10	Number	Powers of 10
0.000001	10^{-6}	1	10^{0}
0.00001	10^{-5}	10	10^{1}
0.0001	10^{-4}	100	10^{2}
0.001	10^{-3}	1000	10^{3}
0.01	10^{-2}	10000	10^{4}
0.1	10^{-1}	100000	10^{5}
		1000000	10^{6}

Expressing numbers in Scientific Notation: Any number written as the product of an integral power of 10 and a number between 1 and 10 is said to be expressed in SCIENTIFIC NOTATION.

The following rules are set down as an aid in expressing large numbers as numbers between 1 and 10 times a power of 10. The rules are stated and examples are given for converting large whole numbers or decimals into scientific notation.

Rule:

In expressing a large whole number as a smaller number usually between 1 and ten, place a decimal point to the right of the last figure of the number and move it to the left until a number to the left of the decimal point is between one and 10. The number of places the decimal point was moved will give the proper positive power of 10.

EXAMPLE:

$637 = 6.37 \times 10^{2}$
$2,700 = 2.7 \times 10^{3}$
$56.33 = 5.633 \times 10^{1}$

$9,628,000 = 9.628 \times 10^{6}$
$5,622.8 = 5.6228 \times 10^{3}$
$873,000 = 8.73 \times 10^{5}$

Rule:

In expressing a decimal as a whole number between 1 and 10, move the decimal point to the right 'l there is a number between 1 and 10. Count the number of places the decimal point was moved this will be the proper negative power of 10.

EXAMPLE:

$0.871 = 8.71 \times 10^{-1}$
$0.0021 = 2.1 \times 10^{-3}$
$0.00000017 = 1.7 \times 10^{-7}$

$0.00078 = 7.8 \times 10^{-4}$
$0.063 = 6.3 \times 10^{-2}$
$0.000029 = 2.9 \times 10^{-5}$

Addition and subtraction of numbers in scientific notation: Numbers expressed in scientific notation can only be added or subtracted if the powers of 10 are the same. For example, 3×10^5 can be added to 2×10^5 to get 5×10^5; however, 3×10^6 cannot be added to 2×10^5 because the powers of 10 are not the same. The number 3×10^6 can be changed to 30×10^5, however, and it can then be added to 2×10^5 to obtain 32×10^5. The answers to problems solved by using scientific notation can be left in the exponential form. In the examples below, however, the answers are converted to the decimal form to aid in understanding this technique.

EXAMPLES:

Add 450,000 and 763,000

$$450{,}000 + 763{,}000 = 4.5 \times 10^5 + 7.63 \times 10^5 = 12.13 \times 10^5 = 1.213 \times 10^6 = 1{,}213{,}000$$

Add 0.00006825 and 0.00000754

$$0.00006825 + 0.00000754 = 68.25 \times 10^{-6} + 7.54 \times 10^{-6} = 75.79 \times 10^{-6} = 7.579 \times 10^{-5} = 0.00007579$$

Multiplication of numbers in scientific notation:

When multiplying numbers written in scientific notation the low of exponents referring to multiplication of numbers raised to a power is applicable. Expressed in general form:

$$A^m \cdot A^n = A^{m+n} \quad (A \neq 0)$$

EXAMPLE:

Multiply 100,000 by 1,000

$$100{,}000 \times 1{,}000 = 10^5 \times 10^3 = 10^{5+3} = 10^8 = 100{,}000{,}000$$

Multiply 25,000 by 5,000

$$25{,}000 \times 5{,}000 = 2.5 \times 10^4 \times 5 \times 10^3 = 2.5 \times 5 \times 10^{4+3} = 12.5 \times 10^7 = 1.25 \times 10^8 = 125{,}000{,}000$$

Multiply 1,800, 0.000015, 300 and 0.0048

$$1{,}800 \times 0.000015 \times 300 \times 0.0048 = 1.8 \times 10^3 \times 1.5 \times 10^{-5} \times 3 \times 10^2 \times 4.8 \times 10^{-3} = 1.8 \times 1.5 \times 3 \times 4.8 \times 10^{3-5+2-3} = 38.88 \times 10^{-3} = 3.888 \times 10^{-2} = 0.03888$$

Division of numbers in scientific notation: When dividing numbers written in scientific notation the law of exponents referring to the division of numbers raised to a power. Expressed in general form:

$$\frac{A^m}{A^n} = A^{m-n} \quad (a \neq 0)$$

EXAMPLE:

Divide 14,400,000 by 1,200,000

$$\frac{14{,}400{,}000}{1{,}200{,}000} = \frac{144 \times 10^5}{12 \times 10^5} = \frac{144}{12} \times 10^{5-5} = 12$$

Divide 75,000 by 0.0005

$$\frac{75{,}000}{0.0005} = \frac{75 \times 10^3}{5 \times 10^{-4}} = \frac{75}{5} \times 10^{3+4} = 15 \times 10^7$$

$$= 150{,}000{,}000$$

Finding the power of a number in scientific notatio: When finding the power of a power which is raising to a power a number expressed in scientific notation to a power the applicable low of exponents is used:

$$(A^m)^n = A^{m \times n} \quad (A \neq 0)$$

EXAMPLE:

Square 15,000

$$(15,000)^2 = (15 \times 10^3)^2 = 15^2 \times 10^{3 \times 2}$$
$$= 225 \times 10^6 = 2.25 \times 10^8$$
$$= 225,000,000$$

Cube 2,000

$$(2,000)^3 = (2 \times 10^3)^3 = 2^3 \times 10^{3 \times 3}$$
$$= 8 \times 10^9$$
$$= 8,000,000,000$$

Square 0.0000075

$$(0.0000075)^2 = (7.5 \times 10^{-6})^2 = 7.5^2 \times 10^{-6 \times 2}$$
$$= 56.25 \times 10^{-12} = 5.625 \times 10^{-11}$$
$$= 0.00000000005625$$

Finding the root of a number in scientific notation: When finding the root of a number raised to a power which is finding the root of a number expressed in scientific notation the applicable low of exponents is used:

$$\sqrt[n]{A^m} = A^{m/n} \qquad (A \neq 0)$$

EXAMPLE:

Find the square root of 25,000,000

$$\sqrt{25,000,000} = \sqrt{25 \times 10^6} = \sqrt{25} \times \sqrt{10^6}$$
$$= \sqrt{25} \times 10^{6/2} = 5 \times 10^3$$
$$= 5,000$$

Find the square root of 144,000,000

$$\sqrt{144,000,000} = \sqrt{144 \times 10^6} = \sqrt{144} \times \sqrt{10^6}$$
$$= 12 \times 10^{6/2} = 12 \times 10^3$$
$$= 12,000$$

TRIGONOMETRY

TRIGONOMETRY is the science of measuring the sides and angles of triangles. In the study of trigonometry, use is made of the fact that a definite relationship exists between angles and their sides. These relationships (called trig functions) have been named and defined. They form the nucleus of trigonometry.

59-33. Definitions

Before trigonometry can be applied to problem solving, some basic definitions must be given.

Angles: An ANGLE is formed when two lines meet at a point. The two lines are called the SIDES of the angle, and the meeting point of the lines is called the VERTEX of the angle. Figure 59-2 shows a graphical representation of an angle.

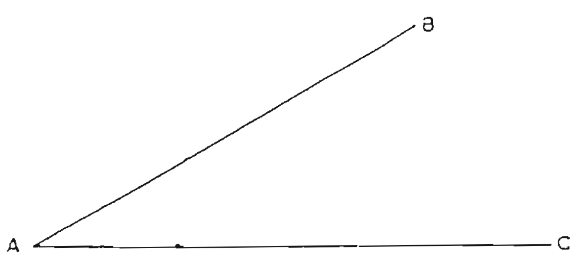


Figure 59-2 - Acute angle.

The symbol used to represent an angle is $\angle$. Frequently, letters of the Greek alphabet are used to represent angles. One of the most widely used Greek symbols is θ, pronounced THETA.

Angles can be generated by a revolving line. If the line AB in Figure 59-3 is rotated about a poin (A), an angle is formed. The magnitude of the angle is given in reference to the STARTING o INITIAL POINT. The dotted line, AC, in Figure 59-3, which could have been a solid line, is callec the LEADING SIDE. The line AB is the TERMINAL SIDE.

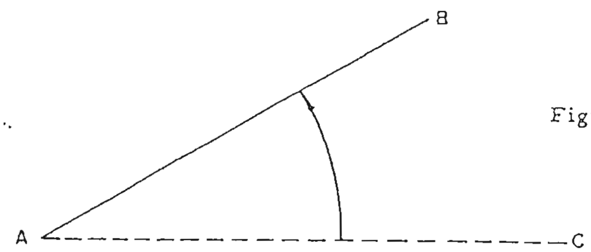


Figure 59-3 - Acute angle.

The magnitude of an angle is generally expressed in DEGREES. If the terminal side of an angle is rotated through a full revolution, it is said to have generated an angle of three hundred and sixty degrees. Numerically, this angle can be represented as 360°. Of course, an angle greater or smaller than 360° can be generated.

Since a line having gone through a complete revolution has also gone through 360°, one degree may be defined as the angle generated when a line has rotated 1/360 of a full revolution. The degree is further divided into MINUTES and SECONDS. One sixtieth of a degree is a minute. One sixtieth of a minute is a second.

EXERCISE 34:

1. How many degrees are there in 1/4 of a revolution, 1/2 revolution, and 3/4 revolution?

2. How many degrees are there in two revolutions, three revolutions, and eight revolutions?

Acute Angle: An ACUTE ANGLE is an angle less than ninety degrees.

Obtuse Angle: An OBTUSE ANGLE is an angle greater than ninety degrees.

Right Angle: A RIGHT ANGLE is equal to ninety degrees.

Negative Angle: A NEGATIVE ANGLE is one which is generated with clockwise rotation of the terminal sides.

Complimentary Angles: COMPLIMENTARY ANGLES are two angles the sum of which is equal to ninety degrees.

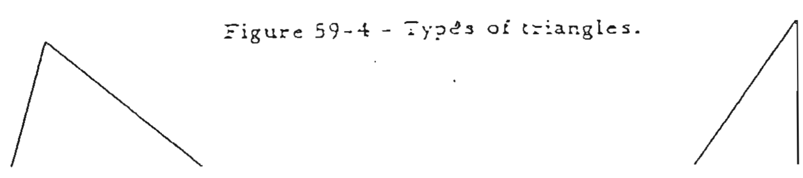
Supplementary Angles: SUPPLEMENTARY ANGLES are two angles the sum of which is equal to one hundred and eighty degrees.

Triangles: A TRIANGLE is a geometrical figure having three sides (sometimes called legs) and three angles. The sum of the angles of a triangle is equal to 180°. Figure 59-4 shows four types of triangles.

The ISOSCELES triangle has two equal sides, the EQUILATERAL triangle has three equal sides, and the SCALENE triangle has no equal sides.

The RIGHT triangle is considered here as a special case because it is important to the study of basic trigonometry.

Figure 59-4 - Types of triangles.



Right Triangle: A right triangle is a triangle which has one ninety degree angle. The trigonometric functions are defined using the right triangle. Figure 59-5 shows a right triangle with its sides and angles labeled.

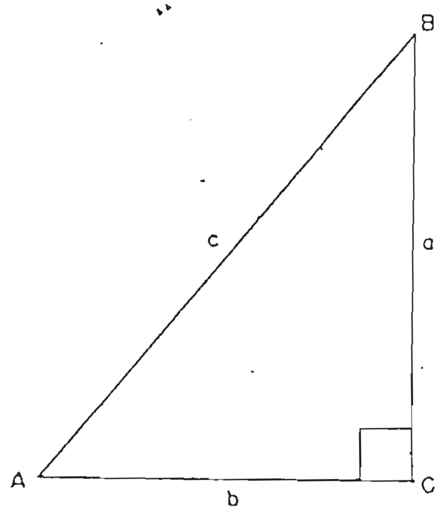


Figure 59-5 - Right triangle.

Side c is called the HYPOTENUSE of the right triangle. The hypotenuse is the longest side of the triangle, and it is opposite the largest angle (90° angle).

The relationship which exists between the sides of a right triangle is described by the PYTHAGOREAN THEOREM.

Pythagorean Theorem: This theorem states that the square of the hypotenuse is equal to the sum of e squares of the other two sides. This relationship may be expressed mathematically as:

$$c^2 = a^2 + b^2$$

To solve for the hypotenuse, the square root of both sides of the equation is extracted.

This gives: $$c = \pm\sqrt{a^2 + b^2}$$

A negative square root has no meaning here. Therefore, the final form is:

$$c = \sqrt{a^2 + b^2}$$

To solve for side b, subtract a^2 from each member of the equation.

This gives: $$b^2 = c^2 - a^2$$

Since the first power of b is desired, extract the square root of both sides of the equation.

Therefore:

$$b = \sqrt{c^2 - a^2}$$

The negative root is ignored because it has no meaning in this application.

Side a may be solved in the same manner.

$$a = \sqrt{c^2 - b^2}$$

A graphical representation of the theorem is frequently shown in the following manner:

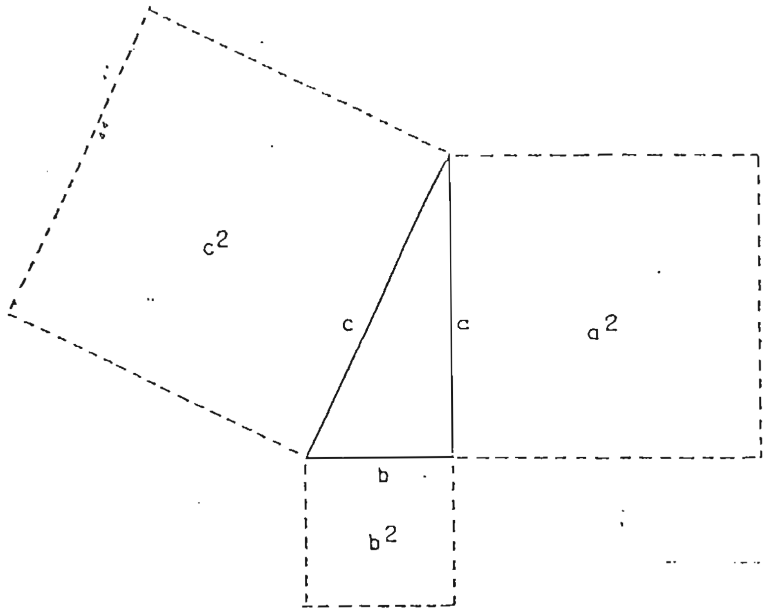


Figure 59-6 - Pythagorean theorem.

The square on the hypotenuse is equal to the sum of the squares on the legs.

EXERCISE 35:

1. One leg of a triangle is twice as long as the other. If the hypotenuse is 10 units long, how long are the two legs?

2. What is the length of the hypotenuse in the following diagram?

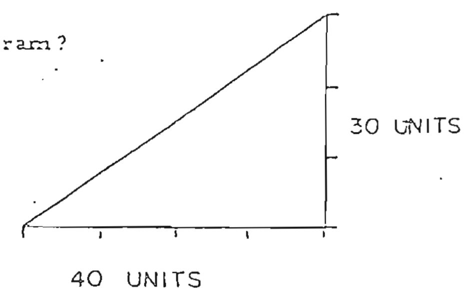


Figure 59-7 - Application of Pythagorean theorem.